points show the constant flow rate case and Eq. (3.3), respectively. Processing of the experimental data of [3] also gives values close to the theory. We note that the results obtained lie above the data of [14], which is evidently due to the difference of geometry of the wetted bodies and to the unsteady nature of the heat transfer process in an experiment conducted on thermally nonconducting materials in the zone behind the blowing section. For these conditions of the isothermal surface from the data reduction we can evaluate the heat flux to the conical surface as a function of the governing parameters of the problem.

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DYNAMICS OF LOW-AMPLITUDE PULSE WAVES IN VAPOR−GAS−DROP SYSTEMS

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The propagation of weak monochromatic waves in vapor and gas suspensions, as well as in gas, vapor, and fluid drop mixtures, was treated in [1-8]. In the present paper we present results on propagation of low-amplitude pulse perturbations in single- and two-component gas−drop systems. An evolution wave-like equation, describing the propagation of linear perturbations in single-component suspensions in the prsence of phase transformations, is obtained and analyzed. Using the fast Fourier transform method, the evolution of a single pulse perturbation in a two-component vapor−gas−drop mixture is calculated. The evolution of interphase friction and phase transformation effects on the wave evolution process are analyzed.

The two-velocity and three-temperature continuum model [9] is used under conditions of acoustic homogeneity of the monodisperse mixture under consideration to investigate a variety of effects. We write down the linearized equations of planar one-dimensional motion in the presence of phase transitions. In a coordinate system in which the unperturbed mixture is at rest the conservation equations of mass, momentum, and energy of the phases are [8]

$$\frac{\partial \rho_1'}{\partial t} + \rho_{10}\frac{\partial v_1'}{\partial x} = -n_0 j_{V\Sigma}, \quad \frac{\partial \rho_V'}{\partial t} + \rho_{V0}\frac{\partial v_1'}{\partial x} = -n_0 j_{V\Sigma}, \quad \frac{\partial \rho_2'}{\partial t} + \rho_{20}\frac{\partial v_2'}{\partial x} = n_0 j_{\Sigma},$$

$$\rho_{10}\frac{\partial v_1'}{\partial t} + \frac{\partial p_1'}{\partial x} + n_0 f = 0, \quad \rho_{20}\frac{\partial v_2'}{\partial t} = n_0 f,$$

$$\rho_{V0}\frac{\partial i'_V}{\partial t} + \rho_{G0}\frac{\partial i'_G}{\partial t} = \alpha_{10}\frac{\partial p'_1}{\partial t} - n_0 q_{1\Sigma}, \quad \rho_{20}\frac{\partial u'_2}{\partial t} = -n_0 q_{2\Sigma},$$

$$q_{1\Sigma} + q_{2\Sigma} = -j_\Sigma l_0, \quad j_{V\Sigma} = j_\Sigma,$$

$$\rho_{10} = \alpha_{10}\rho^0_{10}, \quad \rho_{20} = \alpha_{20}\rho^0_{20}, \quad \alpha_{10} + \alpha_{20} = 1, \quad \alpha_{20} = \frac{4}{3}\pi a^3_0 n_0,$$

$$\rho_{10} = \rho_{V0} + \rho_{G0}, \quad p_{10} = p_{V0} + p_{G0}.$$

(1)

Here ρ, ρ^0, v, and p are the reduced and true density, velocity, and pressure; α is the bulk content; n is the number of particles per unit volume; f is the force exerted by the host phase on an individual drop; $j_{V\Sigma}$ is the diffusion flux of vapor to the surface of the drop Σ; j_Σ is the condensation intensity at the surface of the drop; i, u, and l are the specific enthalpy, internal energy, and vapor formation heat; $q_{j\Sigma}$ is the intensity of heat exchange of the j-th phase with the surface of the drop (j = 1, 2); the subscripts 1 and 2 refer to parameters of the gas and suspended phases; V and G denote parameters of the vapor and gas components of the host phase; the prime superscripts denote perturbation parameters; and the subscript 0 corresponds to the initial unperturbed state. It is assumed that the components of the gas phase are calorically ideal gases. The drops are assumed to be incompressible.

According to the principle of superposition, within linear analysis any acoustic perturbation can be reprsented as the sum of two harmonic waves superimposed on each other. In that case effects related to nonharmonicity of a pulse can be determined as the sum of effects created separately by each of the harmonic components. Therefore, in providing laws of interphase interaction we use relations obtained for harmonic perturbations.

In providing the force interaction of phases we take into account that the basic forces acting on a particle of the disperse phase are Stokes and Bass forces. The expression for the total force is [8]

$$\frac{n_0 f}{\rho_{20}} = \frac{v'_1 - v'_2}{\tau^*_v}, \quad \tau^*_v = \tau_v\left[1 + \frac{1-i}{\sqrt{2}}(\omega\tau_{\mu 1})^{1/2}\right]^{-1},$$

$$\tau_v = \frac{2}{9}\frac{\rho_{20}a^2_0}{\mu_{10}} = \frac{2}{9}\frac{\rho^0_{20}}{\rho^0_{10}}\tau_{\mu 1}, \quad \tau_{\mu 1} = \frac{\rho^0_{10}a^2_0}{\mu_{10}} \quad (\tau_v \gg \tau_{\mu 1}),$$

(2)

where $\tau_{\mu 1}$ is the characteristic time of establishing a quasistationary velocity distribution in the gas phase, τ_v is the Stokes relaxation time of phase velocities, τ^*_v is its complex analog, μ_1 is the dynamic viscosity of the host phase, and i is the imaginary unit.

To determine the dependence of thermal fluxes $q_{j\Sigma}$ (j = 1, 2) on the oscillation frequency ω of harmonic components we use the expressions [5]

$$\frac{n_0 q_{1\Sigma}}{\rho_{10}c_{10}} = \frac{T'_1 - T'_\Sigma}{\tau^*_{T1}}, \quad \frac{n_0 q_{2\Sigma}}{\rho_{20}c_{20}} = \frac{T'_2 - T'_\Sigma}{\tau^*_{T2}},$$

$$\tau^*_{T1} = \frac{\alpha_{10}}{3\alpha_{20}}\tau_{\lambda 1}\eta_1(z_1), \quad \tau^*_{T2} = \frac{1}{15}\tau_{\lambda 2}\eta_2(z_2),$$

$$\eta_1(z_1) = \frac{1}{1+z_1}, \quad \eta_2(z_2) = \frac{5\left[3z_2 - (3+z^2_2)\,\text{th}\,z_2\right]}{z^2_2(\text{th}\,z_2 - z_2)},$$

$$z_j = \frac{1-i}{\sqrt{2}}(\omega\tau_{\lambda j})^{1/2}, \quad \tau_{\lambda j} = \frac{a^2_0}{\varkappa_j}, \quad \varkappa_j = \frac{\lambda_{j0}}{\rho^0_{j0}c_{j0}}, \quad j = 1,2.$$

(3)

Here τ^*_{Tj} is the complex relaxation time of temperature in the j-th phase, determined by the characteristic time $\tau_{\lambda j}$ and the frequency ω; c, λ, and $\varkappa$ are the heat capacity, heat conductivity and temperature conductivity coefficients, and T is the temperature.

The intensity of the vapor diffusion flux $j_{V\Sigma}$ to the phase separation surface on frequency ω is written as [7]

$$\frac{n_0 j_{V\Sigma}}{\rho_{10}} = \frac{1}{\tau^*_p}\frac{p'_V - p'_{V\Sigma}}{p_{10}}, \quad \tau^*_p = \frac{\alpha_{10}}{3\alpha_{20}}\frac{R_V}{R_{10}}(1 - k_{V0})\tau_d\varphi(y),$$

$$\varphi(y) = \frac{1}{1+y}, \quad y = \frac{1-i}{\sqrt{2}}(\omega\tau_d)^{1/2}, \quad \tau_d = \frac{a^2_0}{D_{10}} \quad (|\varphi(y)| \leqslant 1),$$

(4)

where τ_p^* is the complex relaxation time parallel to the vapor motion, determined by the characteristic time τ_d, the oscillation frequency, and the vapor concentration in the gas phase, $k_V = \rho_V/\rho_1$; D_1 is the coefficient of binary diffusion, and R is the gas constant.†

If the phase transformations at the phase separation boundary occur at nonequilibrium, the vapor pressure at the boundary $P_{V\Sigma}$ differs from the saturation pressure $P_{VS}(T_\Sigma)$. The nonequilibrium condensation intensity at the interphase surface is given by the Hertz—Knudsen—Langmuir equation [8]

$$\frac{n_0 j_\Sigma}{\rho_{10}} = \frac{1}{\tau_\beta} \frac{p'_{V\Sigma} - p'_{VS}}{p_{10}}, \quad \tau_\beta = \frac{\alpha_{10}}{3\alpha_{20}} \sqrt{\frac{2\pi}{\gamma_V}} \frac{\gamma_1 C_V a_0}{\beta C_{10}}, \quad p'_{VS} = \left(\frac{dp_{VS}}{dT}\right) T'_\Sigma, \tag{5}$$

where τ_β is the characteristic time of vapor partial pressure matching at the interphase boundary, depending on the value of the accomodation coefficient β; C is the sound velocity; γ is the adiabatic index, and the subscript s refers to parameters at the phase equilibrium line.

The system of equations (1)-(5) is used to investigate acoustic perturbations in rare-gas mixtures with the vapor and with fluid drops.

The investigation of propagation of weak perturbations of arbitrary profile in these media by means of analytic methods is often conducted on the basis of a single wave equation. In this context it is useful to obtain such an equation for the vapor—gas—drop mixture described by the system (1)-(5). In the general case these contributions are quite awkward, therefore we restrict ourselves to the analysis of two specific examples.

The simplest case is that of a gas with solid particles without phase transformations $(k_V = 0)$, treated earlier in [10]. In that case the wave equation for the pressure perturbation can be written in the following form convenient for analysis:

$$\tau_v \tau_T \frac{\partial^2}{\partial t^2} \left\{ \frac{\partial^2}{\partial t^2} p' - C_1 \frac{\partial^2}{\partial x^2} p' \right\} + \frac{\partial}{\partial t} \left\{ (m_1 \tau_T + m_2 \tau_v) \frac{\partial^2}{\partial t^2} p' - \right.$$

$$\left. - C_1^2 (\tau_T + m_3 \tau_v) \frac{\partial^2}{\partial x^2} p' \right\} + m_1 m_2 \left\{ \frac{\partial^2}{\partial t^2} p' - C_e^2 \frac{\partial^2}{\partial x^2} p' \right\} = 0,$$

$$m = \frac{\rho_{20}}{\rho_{10}}, \quad m_1 = 1 + m, \quad m_2 = 1 + m\frac{c_2}{c_{V1}}, \quad m_3 = 1 + m\frac{c_2}{c_{p1}}, \tag{6}$$

$$\tau_T = \tau_{T2} + \frac{mc_2}{c_{p1}} \tau_{T1} = \rho_2^0 c_2 a_0^2 \left(\frac{1}{3\lambda_1} + \frac{1}{15\lambda_2}\right).$$

Here τ_T is the characteristic time of temperature matching between particles and the gas, and C_e is the equilibrium speed of sound in the two-phase mixture, determined by the expression

$$C_e = C_1 \left[\frac{\gamma_e}{(1+m)\gamma_1}\right]^{1/2}, \quad \gamma_e = \frac{c_{p1} + mc_2}{c_{v1} + mc_2} \tag{7}$$

(γ_e is the analog equilibrium adiabatic index for a gas suspension).

It is easily seen that if thermodynamic equilibrium between the mixture phases is established instantaneously $(\tau_V \sim \tau_T \sim 0)$, then the pressure perturbations propagate in the medium with the equilibrium speed of sound C_e. For τ_V, $\tau_T \to \infty$ the interphase interaction is frozen, and the acoustic waves propagate with the frozen speed of sound $C_f = C_1$.

We note that in the absence of interphase friction $(\tau_V = 0)$ Eq. (6) simplifies and agrees with the corresponding equation in [11]. In the absence of particles $(m = 0, \tau_V = \tau_T = 0)$ we have the ordinary wave equation for a gas, when there are no wave dispersion and dissipation.

Consider a suspension with phase transformations. We restrict ourselves to the special case of small mass drop content $(m \ll 1)$, when the effect of interphase mass exchange on wave propagation is most substantial [8]. For a mixture of vapor with drops $(k_V = 1)$, in the absence of quasiequilibrium phase transformations $(T_\Sigma = T_S)$ the evolution equation for pressure

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†The complex times τ_i^* (i = v, T1, T2, p), used in writing f, $q_{j\Sigma}$, $j_{V\Sigma}$, are close to their real quasistationary values τ_i at frequencies $\omega\tau_v < 1$ [3, 8].

waves is[†]

$$\Theta_1 \frac{\partial^3}{\partial t^3} \left\{ \frac{\partial^2}{\partial t^2} p' - C_1^2 \frac{\partial^2}{\partial x^2} p' \right\} + \frac{\partial^2}{\partial t^2} \left\{ \Theta_2 \frac{\partial^2}{\partial t^2} p' - C_1^2 \Theta_3 \frac{\partial^2}{\partial x^2} p' \right\} +$$

$$+ \frac{\partial}{\partial t} \left\{ \Theta_4 \frac{\partial^2}{\partial t^2} p' - C_1^2 \Theta_5 \frac{\partial^2}{\partial x^2} p' \right\} + \Psi_1 \left\{ \frac{\partial^2}{\partial t^2} p' - C_e^2 \frac{\partial^2}{\partial x^2} p' \right\} = 0,$$

$$\Theta_1 = \tau_v \tau_{T1} \tau_{T2}, \quad \Theta_2 = (1 + m)\tau_{T1}\tau_{T2} + (1 + I_1)\, \tau_v \tau_{T2} + (1 + I_2)\tau_v \tau_{T1},$$

$$\Theta_3 = \tau_v \tau_{T1} + \tau_v \tau_{T2} + \tau_{T1}\tau_{T2}, \quad \Theta_4 = \Psi_2 \tau_v + \Psi_3 \tau_{T1} + \Psi_4 \tau_{T2},$$

$$\Theta_5 = \tau_v + \tau_{T1} + \tau_{T2}, \quad \Psi_1 = 1 + m + I_1 + I_2, \tag{8}$$

$$\Psi_2 = 1 + I_1 + I_2, \quad \Psi_3 = 1 + m + I_2, \quad \Psi_4 = 1 + m + I_1,$$

$$I_1 = (\gamma_1 - 1)\left(1 - \frac{\bar{c}_1}{\bar{l}}\right)^2, \quad I_2 = \frac{m\bar{c}_2}{\bar{l}^2}, \quad \bar{c}_1 = \frac{c_{p1}}{\gamma_1 R_{10}}, \quad \bar{c}_2 = \frac{c_2}{\gamma_1 R_{10}}, \quad \bar{l} = \frac{l}{C_1^2}.$$

Here C_e is the equilibrium speed of sound (7) in the gas and drop mixture with index $\gamma_e = [1 + (\bar{c}_1 + m\bar{c}_2 - 2\bar{l})/(\gamma_1 \bar{l}^2)]^{-1}$. If one of the relaxation processes of the interphase interaction with characteristic time τ_i (i = v, T1, T2) is absent, then $\Theta_1 = 0$ and the order of Eq. (8) is reduced. Thus, in studying propagation of waves whose harmonic components have frequencies $\omega\tau_{T2} \ll 1$ the effect of temperature inhomogeneity inside the drop (the difference between T_2 and T_Σ) can be neglected, and in Eq. (8) one can take $\tau_{T2} = 0$, i.e., $\Theta_1 = 0$.

Equations (6) and (8) describe propagation of linear waves in suspensions in both the positive and negative directions of the coordinate x. Their analytic solution encounters large difficulties. In this context we use the method of slowly varying profiles [12], substantially simplifying the solution of the problem and valid for dissipative systems, when the attenuation decrement per wavelength $\sigma = 2\pi K_{**}/K$ (K, K_{**} are the real and imaginary parts of the complex wave number K_*) is small ($\alpha \ll 1$) for all harmonic components of the perturbation.

For gas suspensions, in the absence of phase transformations the maximum value of the coefficient σ is achieved at frequencies $\omega\tau_v \sim \omega\tau_T \sim 1$. In that case the maximum value is $\sigma_{max} \sim m\pi/2$, and for low particle mass content (m $\ll$ 1) the coefficient σ is small in the whole frequency region under consideration $\omega\tau_v < 1$.

In the case of suspensions with phase transitions the coefficient can have two maxima, realized at frequencies $\omega\tau_v \sim m$ and $\omega\tau_v \sim 1$ [8]. The value of the first maximum of σ can also be substantial for drop content m $\ll$ 1 [8]. In that case, for smallness of σ it is necessary to impose additional restrictions on the frequencies of harmonic components; more precisely: $\omega\tau_v \ll$ m.[‡]

Within the assumptions made we investigate the propagation of perturbations traveling to the right. We transform to the accompanying coordinate system, and introduce the new variables [12] $\eta = t - x/C_1$, $\xi = \mu x$. As a result of the transformation each of the equations (6), (8) transforms to an equation of the type of the thermal conduction equation

$$\partial p'/\partial x = A_j \partial^2 p'/\partial\eta^2 \quad (j = G, V) \tag{9}$$

with a known general solution [13]. Here A_j (j = G, V) are coefficients characterizing the absorption of acoustic waves.

For a mixture of gas with particles without phase transformations, for the coefficient A_G we have

$$A_G = \frac{m}{2C_1}\left\{ \tau_v + (\gamma_1 - 1)\frac{c_2}{c_{p1}}\,\tau_T \right\} \quad (\omega\tau_v < 1), \tag{10}$$

[†]The nonequilibrium of interphase mass exchange will affect wave propagation only at quite high frequencies of harmonic components, when [8]

$$\frac{1}{\beta}\left(\frac{\lambda_2}{\lambda_1}\right)\left(\frac{C_1^2}{l}\right)\left(\frac{L}{a}\right)(\omega\tau_{\lambda 2})^{1/2} \gtrsim 1, \quad L = \frac{\varkappa_1}{C_1}.$$

[‡]We note that for the restrictions considered on frequency, due to the smallness of drop mass content m the dispersion in the speed of sound can be neglected, and one can take $C_e = C_1$.

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while in the case of vapor with drops in the presence of interphase mass exchange

$$A_V = \frac{1}{2C_1} \{ m\tau_v + I_1\tau_{T1} + I_2\tau_{T2} \} \quad (\omega\tau_v < m),$$

$$I_1 = (\gamma_l - 1)(1 - \overline{c_l/l})^2, \quad I_2 = \overline{mc_2/l}. \tag{11}$$

We point out that, as a rule, $\tau_{T2} \ll \tau_V$, τ_{T1}; $I_2 \ll m$, I_1, therefore the last term in (11), related to the temperature inhomogeneity inside the drop, can be neglected due to its smallness.

We note that expressions (10), (11) for the coefficients A_j (j = G, V) agree, due to the restrictions imposed on the frequencies, with the corresponding low-frequency asymptotic expressions for the attenuation coefficients $K_{**}(\omega)$ [6]. For a known dependence $K_{**}(\omega)$ this makes it possible to write down an equation of type (9) for the general case of a two-component, two-phase mixture of gas, vapor, and fluid drops. The expression for the coefficient A_m, following from the low-frequency asymptotic expression for $K_{**}(\omega)$ [6], is

$$A_m = \frac{\tau_v}{2C_1} \left\{ m + \frac{3}{2} \mathrm{Pr}_1 (\gamma_l - 1) \left[m\Lambda_1 + k_V \left(\Lambda_2 + \frac{1}{m} \Lambda_3 \right) \right] \middle/ \Lambda_4 \right\} \quad (\omega\tau_v < m), \tag{12}$$

where the coefficients Λ_j (j = 1,...,4) are determined from

$$\Lambda_1 = (\overline{c_2}B_1)^2, \quad \Lambda_2 = \left[1 - \mathrm{Le}_1 - (h - \mathrm{Le}_1)k_V - \frac{\tau_\beta}{\tau_{T1}} \right] (\overline{l}^2 - bh\overline{c_1}^2) \gamma_l \overline{c_2} b,$$

$$\Lambda_3 = \left[\gamma_l \overline{l}^2 k_V + \left[(1 - k_V) \mathrm{Le}_1 + \frac{\tau_\beta}{\tau_{T1}} \right] b\overline{c_1} \right\} B_2, \quad \Lambda_4 = [\gamma_l \overline{l}^2 k_V + \overline{c_1}B_1]^2, \tag{13}$$

$$B_1 = (1 - hk_V)b, \quad B_2 = \gamma_l(\overline{l} - h\overline{c_1})(\overline{l} - b\overline{c_1}),$$

$$h = 1 + (1 - k_V)(R_V - R_G)/R_{10}, \quad b = R_V/R_{10}, \quad \mathrm{Le}_1 = \varkappa_1/D_1.$$

We note that the relationship for A_m (12) agrees with the expressions for A_G (10) and A_V (11) for the limiting values $k_V = 0$ and $k_V = 1$, $\tau_\beta = 0$, respectively. The first term m in the brackets of expression (12) determines the contribution of interphase friction. The relations containing the components Λ_j (j = 1,...,4) characterize interphase heat and mass transfer. In that case, due to the low drop mass content (m $\ll$ 1) at vapor concentrations k_V > m the term with Λ_3 is predominant due to the factor 1/m. Thus, the attenuation of long-wave perturbations considered with frequency harmonics $\omega\tau_v$ < m in aerosols with mass exchange in a wide region of variation of vapor concentration $m < k_V \leqslant 1$ is basically determined by interphase heat and mass transfer. It must also be emphasized that if the coefficient A_G (10) is directly proportional to the particle mass content m for mixtures of gas with solid particles, then for suspensions with phase transformations the dependences $A_V(m)$ (11) and $A_m(m)$ (12), (13) are more complicated functions of m.

Due to the assumptions made the attenuation of perturbations at distances of the order of the wavelength is low. At large distances, however, the contribution of dissipative effects accumulates and becomes substantial. According to the general solution of Eq. (9) [13] the behavior of an isolated pulse at large distances is described by the universal asymptotic expression

$$p'(x, \eta) = \frac{\exp\left(-\eta^2/4A_jx \right)}{\sqrt{4\pi A_jx}} \int p_0'(\xi) \, d\xi, \quad j = G, V, m, \tag{14}$$

where p_0' is the initial perturbation at the point x = 0. The solution (14) corresponds to a pulse of Gaussian shape. Due to the predominant absorption of high-frequency spectral components the width of curve (14) increases proportionally to $(A_jx)^{1/2}$, in which case the wave amplitude decreases as $\sim 1/(A_jx)^{1/2}$.

Consider the more general case of substantial attenuation of weak pulse perturbations in vapor–gas–drop systems.† According to Fourier analysis representations, an arbitrary space–time pulse can be represented as an integral [12]

$$p(x, t) = \int_{-\infty}^{\infty} P(\omega) e^{i[K_*(\omega)x - \omega t]} d\omega, \tag{15}$$

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†Several results of investigating pulse evolution in a bubbly fluid were given in [14].

where the spectral function P(ω) is found from the given initial signal:

$$P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p(0, t) e^{i\omega t} dt. \tag{16}$$

Relationship (15) determines the inverse Fourier transform P(ω), while (16) determines the direct Fourier transform function p(0, t).

Due to the awkwardness of the dispersion dependence $K_*(\omega)$ [8], the analytic determination of the function p(x, t) by Eq. (15) with account of (16) is quite difficult. In this connection it is advisable to find p(x, t) numerically, for which purpose we use methods of the discrete Fourier transform, representing the pulse p(x, t) as a finite sum of harmonic waves:

$$p(x, t) = \sum_{m=0}^{L-1} z_m \exp[K_*(\omega_m) x - i\omega_m t]. \tag{17}$$

For x = 0 we have

$$p(0, t) = \sum_{m=0}^{L-1} z_m \exp(-i\omega_m t). \tag{18}$$

According to (17), to determine p(x, t) it is necessary to find the coefficients z_m, in which case relation (18) must be satisfied. To find the coefficients z_m and determine p(x, t) by computer we use the fast Fourier transform (FFT) algorithm [15-17], allowing one to reduce substantially the number of arithmetic operations required.

Expanding the initial pulse in a discrete Fourier series and combining its harmonic components in the new position, one obtains the transformed pulse formed under the actions of dispersion and dissipation. Several results of calculating the propagation and attenuation process of weak pulses of pressure perturbations in a mixture of air with vapor and water drops of radius $a = 2 \cdot 10^{-6}$ m at host phase pressure $p_1 = 0.1$ MPa are illustrated in Figs. 1 and 2. The vapor concentration in the gas phase was $k_V = 0.1$ ($T_0 = 327$ K), with the accommodation coefficient being β = 0.04. The initial perturbation (primed lines) created at the boundary of the two-phase mixture was given in the form $p(0, t) = \exp[-((t - t_*)/N)^2]$. The pulse duration was ~$5 \cdot 10^{-3}$ sec, and the frequencies of its fundamental harmonics were located in the frequency interval $\omega\tau_V \ll 1$, i.e., the pulse considered was quite long.

The calculations were carried out by means of the dispersion relation of [8] with the use of FFT subroutines [16]. The number of harmonics L was selected from the condition of pulse motion without distortion in the absence of wave dispersion and dissipation ($K_* = \omega/C_1$). Accuracy control of the calculations was realized by recalculation with doubling the number of harmonics.

The evolution pattern of a weak pressure pulse in vapor—gas—drop mixtures with various mass contents of the condensed phase m is shown in Fig. 1 by solid lines (the dashed—dotted lines are the pulse evolution at frozen mass exchange β = 0). The calculated profiles (oscillograms) correspond to distances 4 and 8 m from the pulse minimum positions.

The propagation of low-amplitude perturbation pulses, represented in the form of superpositions of monochromatic harmonics, occurs according to the propagation laws of weak monochromatic waves [8]. In this context and according to the results obtained above for monochromatic waves the attenuation of long-wave pulses in gas suspensions with phase transformations depends nonmonotonically on the mass content of the suspended phase m. Thus, for m = 0.1 the attenuation is not only higher than for m = 0.01, but also higher than for m = 1.0. The interphase mass exchange can strongly affect the perturbation attenuation. For m = 0.1 the contribution of mass exchange to pulse attenuation is substantial (compare the solid and dashed—dotted curves of Fig. 1). As to the pulse propagation rate, for the thermophysical medium parameters considered it is practically independent of the presence or absence of mass exchange. If this drop mass content is sufficiently high (m = 1), the presence or absence of mass exchange affects weakly both the propagation rate and attenuation coefficients of weak pulses (the dashed—dotted lines practically coincide with the solid ones). The attenuation of weak pulse perturbations in gas suspensions without phase transformations is proportional to the particle mass content m, and always increases with increasing m.

The effect of interphase friction on the evolution process of low amplitude pulse perturbations in aerial fog is illustrated in Fig. 2 (dashed—dotted lines) for various drop
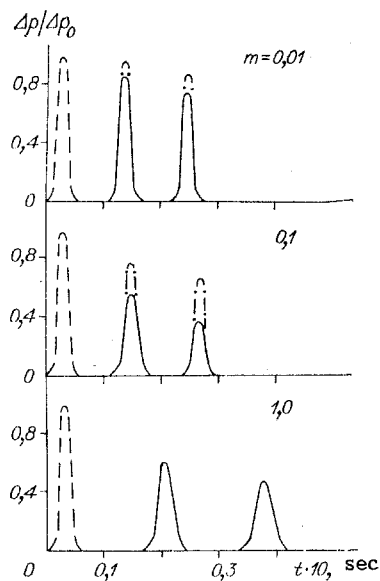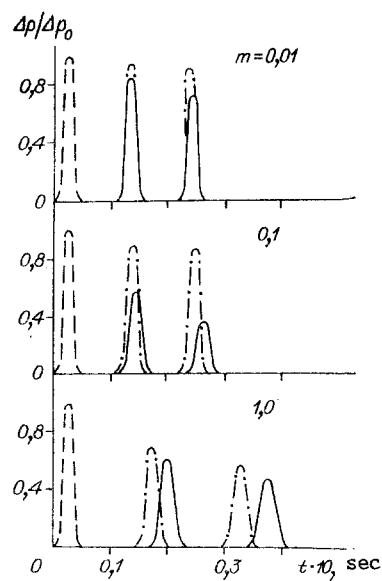
Fig. 1

Fig. 2

mass contents. It is seen that most of the effect on attenuation of long-wave pulse pertur-
bations in aerosols with phase transformations is due to interphase heat and mass transfer.
For increasing mass content of the condensed phase up to values m ~ 1 the effect of inter-
phase friction on pulse attenuation increases, while the effect of interphase heat and mass
transfer on wave propagation remains, nevertheless, substantial.

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A STATEMENT OF THE PROBLEM OF THERMAL DIAGNOSTICS FOR A "PROTECTIVE" LAYER

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By the problem of diagnostics we understand the task of determining the characteristics of a body from information obtained by experiment about physical fields which arise in it under the influence in a special way of assembled external effects [1, 2]. The operating conditions for a number of articles connected with the combined effect of irradiation, force and thermal loading, strong magnetic fields, and corrosive media necessitate provision of the required safety. Here considerable attention should be devoted to studying the destructive effect of these actions on material characteristics both within the scope of periodic monitoring of the condition of the operating structure, and within a schedule of laboratory tests on the materials developed [3].

In the present work the task is considered of determining changes in the specific heat capacity and thermal conductivity coefficient of a material (as a result of accumulation of microdamage) for a weak curvilinear "protective" layer from the results of thermocouple measurements of temperature at its accessible surface, and at the same time the other surface is subject to the destructive effect of a corrosive medium. The possibility is studied of determining the change in layer thickness (which may also occur as a result of the effect of destructive factors) and clarification of the corrosive medium temperature. In mathematical scheme the problem relates to the type of reverse problems of mathematical physics [4].

1. According to [5] spreading of heat in a nonuniform curvilinear layer $\Omega = \{(x_1, x_2, x_3)|-\infty < x_1,\ x_2 < \infty,\ \gamma_1 \leqslant x_3 \leqslant \gamma_2\}$,

$$0 \leqslant \gamma_1 < \gamma_2 \leqslant H - \text{const},\quad \gamma_1 = \gamma_1(x_1, x_2),$$
$$\gamma_2 = \gamma_2(x_1, x_2,\ t) \in C^1$$

(Fig. 1) may be described by the relationships

$$C_\varepsilon \theta^{\cdot} - (K\theta_{,i})_{,i} = f;\tag{1.1}$$

$$(a\theta + bn_i\theta_{,i})|_{x_3=\gamma_1} = p(x_1, x_2, t),\tag{1.2}$$
$$n_k = \gamma_{1,k}(1 + \gamma_{1,1}^2 + \gamma_{1,2}^2)^{-1/2}.\quad n_3 = (1 + \gamma_{1,1}^2 + \gamma_{1,2}^2)^{-1/2},\ k = 1,\ 2;$$

$$\theta|_{x_3=\gamma_2} = q(x_1, x_2, t);\tag{1.3}$$

$$\theta|_{t=0} = \varphi(x_1, x_2, x_3).\tag{1.4}$$

Here specific heat capacity C_ε and the thermal conductivity coefficient for the layer K depend on spatial variables $x = (x_1, x_2, x_3)$, and relative temperature θ is a function of x and time t. A period means partial derivative with respect to time, the index after a comma is derivative with respect to the corresponding coordinate, and everywhere k = 1, 2, i = 1, 2, 3, m = 1, 2,.... Summing is carried out for a repeated index (if not stipulated to the contrary). Boundary condition (1.2) is the condition of heat exchange at the outer (accessible) surface of the layer $x_3 = \gamma_1$. Condition (1.3) is given at the inner (in contact with some corrosive medium of temperature q) surface $x_3 = \gamma_2$. This surface is assumed to be inaccessible for performing direct measurements, and in view of the effect of various factors its geometry may change with the passage of time, i.e., $\gamma_2 = \gamma_2(x_1, x_2, t)$.

The diagnostic problem considered by us involves determining specific heat capacity $K(x)$ and thermal conductivity coefficient $C_\varepsilon(x)$, and also in clarifying the geometry of the layer [functions $\gamma_2(x_1, x_2, t)$ and corrosive medium temperature $q(x, t)$ from two problems of the form (1.1)-(1.4) with different conditions for initiating thermal processes [after substituting $\{\theta, f, p, \varphi\}^k \to \{\theta, f, p, \varphi\}$ in (1.1)-(1.4)] from supplementary information